Higher Order Thinking and Mathematics

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In recent times higher thinking skills have received much attention. In this paper a definition of higher thinking is proposed and their importance in non routine algebra problem solving is discussed. A rationale for the author's doctoral study is presented followed by a brief description of the study itself. Some preliminary results are also included.

Wundt (1924) identified two types of thinking namely, associative and intellectual. Associative thinking is unguided thinking in which thoughts wander while intellectual thinking is more constant and regulated by desire or design. It can be argued that all intellectual thought is aimed at problem solving. In other words, a goal exists and some procedures are executed to change the initial state into the goal state. However, students in mathematics classrooms commonly experience routine problems that require mere recall of procedures. As a result, they fail to engage much intellectual thought. Another aspect of mathematical problem solving is non routine problem solving, that is, problems that have no obvious solution and therefore require the engagement of so-called intellectual thought. Potter, Whimbey and Lochhead (1991) labelled this type thinking as critical thinking. Recent assessments indicate students perform poorly in multistep and non routine algebra problems (Dossey, Mullis, Linguist & Chambers, 1988). It seems that more research is needed in non routine problem solving. In this paper, definitions of intellectual thought are examined and metacognition and critical thinking literature are reviewed by arguing a case for their importance to algebraic problem solving. A rationale is developed for the author's doctoral study. This is followed by a brief description of the author's study including some preliminary results of the study.

Background Literature

Currently, higher order thinking has received some attention in mathematics education (Resnick, 1987). Higher order thinking is a vague term but a definition is developed in this paper. Dewey (1916) defined thinking as an experience. He argued that "reflective experience" involved:

> (i) a careful survey (examination, inspection, exploration, analysis) of all attainable considerations which will define and clarify the problem at hand;

(ii) a consequent elaboration of the tentative hypothesis to make it more precise and more consistent, relating it with a wider range of facts;

(iii) taking one's stand upon the projected hypothesis as a plan of action which is applied to the existing state of affairs: doing something overtly to bring about the anticipated result, and thereby testing the hypothesis.

Dewey (1916) argued that the above features make reflective experience markedly different from trial and error type thinking, labelled earlier as associative thinking. Polya's (1962) heuristics such as understand the problem, develop a plan, carry out the plan and check, are similar to reflective experience. Implicit in Dewey (1933) and Polya's (1962) ideas are thinking skills such self regulation, analysis and evaluation. Bloom (1956) and his colleagues identified analysis, synthesis and evaluation as important and labelled them higher order objectives. Resnick (1987) argued that while it was difficult to define higher order thinking it was nevertheless possible to recognise them when they occur. She considered higher order thinking to be non algorithmic, complex, frequently yielding multiple solutions each with cost and benefits. Also, she noted higher thinking involves nuanced judgements and interpretations, application of multiple criteria, uncertainty, self regulation, imposing meaning, and considerable effortful work. Such thinking skills are important in mathematical learning and problem solving. For example, often students fail to transfer mathematical skills and it can be argued that higher thinking skills can facilitate transfer of knowledge (Paul, 1990).

A number of common themes are observed in the description above. For example, they describe some complex situation that involves a degree of uncertainty, thus requiring the solver to interpret and judge as well as impose structure on the situation. It can be argued that higher order thinking involves three aspects, namely, critical thinking, self regulation of thought and motivational tendencies to sustain the considerable mental work involved. In the next few sections these variables are discussed further.

Metacognition

A set of closely related is called metacognition which comprises cognitions about cognitions or thinking about one's own thinking and thus it belongs to higher order cognitions. There exists some evidence which suggests that students abilities to use these skills can ultimately determine whether they succeed on non routine tasks (Narode, 1987). In non routine algebra problem solving, students need to assess, decide and monitor the consequences of using algebraic skills or other more general solution strategies, such as trial and error.

Flavell (1976) defined metacognition in terms of knowledge and control. Metacognition includes awareness of self actions, reflections on a stream of processing and decision making based on these actions. Later, Flavell (1987) categorised metacognition as metacognitive knowledge and metacognitive experiences. Metacognitive knowledge includes variables such as person, task and strategy. Person variables includes all that one knows about self, and others in relation to self. Task variables refers to knowledge and beliefs about tasks while strategy variables include knowledge and beliefs about strategies that control or supervise a problem solving process. On the other hand, metacognitive experience refers to feelings and emotional that are usually not addressed by other models. Metacognitive experiences frequently refer to feelings experienced after self-evaluations. For example, students may feel proud, happy, sad, disappointed and so on.

More specifically, person variables account for a person's beliefs about self and others, including their beliefs regarding problem solving performances. Often beliefs cause misconceptions and errors, yet some beliefs are important and helpful. For example, while analysing students protocols, Silver (1982) found a number of beliefs that facilitate problem solving. For example, there must be more than one answer, there may be a more precise or short way of solving the problem and so on.

Task variables is metacognitive knowledge about how tasks may affect problem solving process. For example, students may not critically examine text information if they believe algebra tasks do not make sense. Also, students may hold beliefs relating to solution methods of algebra word problems and resort to "key word" solution strategy (Narode, 1987). Using protocol methods, Schoenfeld (1983) found that students often possessed relevant knowledge about a problem but failed to utilize it in solving the problem. For example, "they had the means to solve the problem within their reach, but did not call upon them. They did not even think to call upon them!" (Schoenfeld, 1983, p. 371). In this situation, the students failed to invoke relevant strategies due to their incorrect beliefs about proof problems.

Strategies are heuristics that help reduce complexity in problem solving. They account for the processing, monitoring and regulating of information. For example, in computer problem solving, students practise not only debugging of programs but also decomposition of problems into explicit steps. These two processes are metacognitive in nature and seem to be responsible for the successes claimed by Clement, Lochhead and Soloway (1980) in a study to remediate the "reversal" error in algebra. Reversal error occurs when students reverse the labelling of variables in an equality relationship. Knowledge and beliefs about strategies are also included in metacognitive knowledge. For example, students frequently engage more general strategies such as trial and error because they believe these to be simpler (Schoenfeld, 1985). Yet, others use a step by step methodical approach, a strategy they believe important in problem solving. In this manner, Flavell (1987) argued that personal, task and strategy may interact.

Evidently, only a few studies have documented its use in algebraic problem solving (Montague & Bos, 1990). Not surprisingly, some researchers have argued for more research in the area. For example, Reeve and Brown (1984) suggested relationships between metacognition and motivation should provide a focus for further study. Also, Garofalo and Lester (1985) noted a lack of research relating metacognitive knowledge and beliefs to performance in mathematics. Another equally important but lowly researched area in mathematics is critical thinking.

Critical Thinking

Critical thinking has been identified as an essential goal of education (Paul, 1990). Although a number of definitions exist in literature, Riesenmy, Mitchell and Hudgins (1991) and Paul's (1990) definitions are used in this paper. Riesenmy, Mitchell and Hudgins (1991) stated that critical thinking involves purposeful actions to exhaustively survey the given information to isolate knowns and unknowns; and assessing the information given or answers generated against some criteria; while Paul (1990) defined critical thinking in terms of five macro processes which are:

(i) Socratic Questioning-questioning ourselves or others so as to make explicit the salient features of our thinking. (eg. Are there alternatives? What assumptions are we making? What concepts are we using? are they appropriate? Do we grasp them? What are the implications of our reasoning?);

(ii) Conceptual Analysis-any problematic concepts or uses of terms must be analysed and their basic logic set out and assessed. (eg. Have we done so?);

(iii) Analysis of the Question at Issue-whenever one is reasoning one is attempting to settle a question at issue. But to settle a question, one must understand the kind of question it is. Different questions require different modes of settlement. (eg. Do we grasp the precise demands of the question at issue?);

(iv) Reconstructing Alternative Viewpoints in their Strongest Forms; and

(v) Reasoning Dialogically and Dialectically.

Implicit in these macroprocesses are microprocesses that are identifiable when students are using socratic questioning, their minds make a variety of moves that are made explicit by the use of analytic terms such as: claims, assumes, implies, infers, concludes, is relevant to, is irrelevant to and so on (Paul, 1990). Importantly, it is then possible to identify when critical thinking occurs. Riesenmy, Mitchell and

Hudgins' (1991) definition and Paul's (1990) first three macro processes are particularly relevant to high school word problem solving. In support, Potter, Whimbey and Lochhead (1991) argued that "the ability to solve multistep word problems of any type is a primary expression of what researchers regard as the essence of critical thinking, a process requiring reflective thought to clarify meaning and construct relationships" (p. 6).

While studies on critical skills in algebraic problem solving are scarce, some evidence is available in general problem solving mainly through some early work (see Riesenmy, Mitchell and Hudgins, 1991). More recently, Riesenmy, Mitchell and Hudgins and Cooper, Baturo and Smith (1993) found that by analysing students protocols and relying on interviewers' perceptions, students who were taught thinking roles such as definer, strategist, monitor and challenger performed better on everyday tasks than control children. Similar support has been reported by Montague and Bos (1990) in their study of eighth grade mathematical word problem solving and Potter, Whimbey and Lochhead's (1991) study on word problem solving. Also, there exists some evidence that suggests that such general skills can be transferred to content areas such as mathematics (Adey & Sheyer, 1990).

Affective Variables

Affective variables such as motivation has been a rather neglected area in algebraic problem solving. However, Resnick's (1987) mentioned that considerable mental effort is required during critical thinking. For example, expert/novice studies indicate that experts' "motivation to succeed" levels differ significantly from that of novices. It can be argued that "motivation to succeed" may be a crucial factor in the engagement of appropriate metacognitive and critical thinking strategies. Moreover, Rosenberg (1986) defined affective awareness as a critical thinking skill. That is, being aware of self emotions. It seems that higher thinking requires one to be alert to ideas, willing to take risks as well as possessing positive attitudes towards the task. For example, Candy's (1991) spoke of critical thinking as "critical spirit" or "attitude". In other words, engaging a habit of using critical skills or a set of attitudes and character trait.

Research Questions

There is a paucity of research studies on higher thinking and algebra. Importantly, reported studies in this area suggest further examination. For example, Schoenfeld's (1985) findings in geometry, suggests managerial factors as a cause of failure but novices fail even when they functionalize metacognitive strategies. Evidently, affective factors influence critical thinking so there is a need to examine how, for example, motivation and higher thinking interact and how they both affect algebra problem solving. Additionally, how beliefs influence non routine algebraic problem solving needs further research. While recognizing the influence of motivational tendencies in expert performances, past studies have not examined motivational, higher thinking and content knowledge within a given study. A simultaneous examination may provide important insights about the nature of interactions between such variables. Important frameworks are available with which these variables may be examined. For example, information processing provides a theoretical framework for studying cognitive and metacognitive processes because it can address the complex interplay between such strategies as students solve problems. In addition to studying metacognitions, Flavell's (1987) model provides a framework with which external variables such as contextual and affective factors can be studied.

The Study

The lack of research in general led to this exploratory study which examines three variables during algebraic problem solving; namely, higher order thinking skills, algebraic content knowledge and achievement motivation. Approximately 120 Year 11 students were tested out of which 16 were selected based on their scores; and each labelled, HHH, HHL, HLH and so on in the three areas respectively (H represent High score; L represents Low score) Further, 3 experts were selected, a Year 12 Australian Olympiad winner, a pure mathematics doctoral student and a senior lecturer in mathematics education. Non routine as well as a few routine word problems were selected using a pilot study. The selected subjects were clinically interviewed and requested to talk aloud while they solved selected problems. All interviews were videod totalling more than 40 hours. The tapes were transcribed and codes developed to analyse the protocols. Analysis of the data includes the coding of protocols, timing of problem solving episodes and counting of codes.

Preliminary Results and Discussion

Successful subjects demonstrated extensive qualitative processing during the initial stages. For example, they performed actions to interpret the situation, identify the relevant information, identify assumptions, relate text to prior knowledge and so on. Moreover, they enjoyed challenges and persisted even when failure occured. In many cases reflective elaborations of the text data led the subjects to generate relationships that may or may not have involved algebra, yet further logical manipulations ultimately led the subjects to apply suitable procedures. As expected, experts used various higher thinking while solving the non routine word problems. Importantly, HHH subjects behaved in much the same manner. The similarities between expert performances and HHH performances appear significant. For example, it suggests that high school students are capable of expert-like performances even when they lack much of the experience and knowledge of experts. The HHL subjects also behaved similar HHH and the experts however, they were not as committed, often searching for short cuts that led them to make careless errors. These subjects did not appear to enjoy such problem solving yet performed better than other groups except experts and

HHH. The HLH students also utilised many strategies mentioned above and performed much better than HLL, LHL, LLH and LLL subjects. The lack of algebraic knowledge was somewhat compensated by their extensive use of higher thinking skills but their lack of algebraic knowledge and skills was frequently a reason for failure. This highlights the importance of domain specific knowledge in problem solving. On the other hand, HLL subjects did not perform markedly better than LLH subjects. The HLH and HLL results suggest that achievement motivation may be an important factor in functionalizing higher skills. While LHL subjects had good algebraic content knowledge they performed poorly in relative terms. They succeeded only in the simpler, more routine type word problems. These students did not appreciate this type of problem solving because of the concentration involved and frequently failed to persist. In general, LHL students did not apply higher order thinking strategies as expected. On the other hand, LHH subjects performed better than HLL, LHL, LLH and LLL. They relied on algebra in most situations but this presented difficulties during more complex problems that needed higher skills to analyse and evaluate situations. Although they persisted during such problems frequently a lack understanding of the crucial relationships suggested in the word problems caused failure.

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